

Modeling and identification of irrigation station with sprinkling: Takagi- Sugeno approach

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Abstract—the spray under pressure is an effective save on water. This task should be automated and controlled in order to limit the water waste and the facilities of damages. For this reason, it's necessary to find a mathematical model describing the irrigation process. In order to facilitate this step the Takagi-Sugeno fuzzy model is the best approaches of nonlinear systems representation. Various techniques are used in the literature of such systems; the clustering technique is one of the best solutions. In this paper, we'll model the irrigation station with the T-S algorithm and use the fuzzy c-means (FCM) algorithm and present the results of simulation

Keywords— Irrigation Station; T-S modeling; FCM algorithm; clusters.

I. INTRODUCTION

The development of a mathematical model making it possible to represent "as well as possible" the dynamic behavior of a complex real process represents a very important problem in the practical world. In recent years, and with the evolution of technology, a significant effort has been given to modeling, identification and control of such systems. The Takagi-Sugeno fuzzy model ([1]; [2]) is one of the best approaches to the representation of such a process. Indeed, the T-S fuzzy model can approximate highly nonlinear system into several locally linear subsystems interconnected. The identification problem in the T-S fuzzy model can be summarized in two steps: structure identification and parameter estimation. Several techniques were developed to conclude the modeling of these systems: we quote primarily the neuro-fuzzy technique [3] and clustering technique ([4]; [5]; [6]; [7]; [8]; [9]). Indeed Several researchers have noticed that a nonlinear system can be approximated by the sum of several linear subsystems. Method of clustering proves to be an interesting technique for identification and the modelisation of the nonlinear systems. Indeed, this technique consists in approximating the total nonlinear system by a vague model of Takagi-Sugeno type. In this case, each cluster represents one fuzzy rule of Takagi-Sugeno. The number of clusters is fixed by an expert according to the type and the performances of application considered. By consequent to each cluster one correspond homogeneous zone of operation such that is defined in the form of a linear local model. We are interested to model and identify a nonlinear system by the fuzzy logic approach such as Takagi-Sugeno (T-S) approach. The latter, uses modeling containing linguistic rules to obtain the model of system outputs. Initially, we present the fuzzy logic approach design, we gives an outline on the first two models. Then, we

detail (T-S) model, uses the method of fuzzy coalescence for the identification of the nonlinear systems by the fuzzy C-means (FCM) algorithm. We will in addition present tests of validation of (T-S) model. Then, we will give the results of identification and modeling of the station of irrigation by sprinkling.

The remainder of this paper is described such as the following section. In the first section we have describe the Fuzzy coalescence algorithms. Secondly, we spend to detail the FCM algorithm step by step. Finally, we finished by application of FCM algorithm to the irrigation station by sprinkling located in the laboratory shown in the Fig 1. After identification and modelisation with FCM algorithm it is necessary to validate our simulation results (model mathematic of our pumping station) with Root Mean Square Error test (RMSE) and the Variance accounting for test (VAF).

The French company LEROY- SOMMER makes available to researchers an irrigation station (Fig 1) with sprinkling but with practical constraints existing in the real irrigation stations [16].

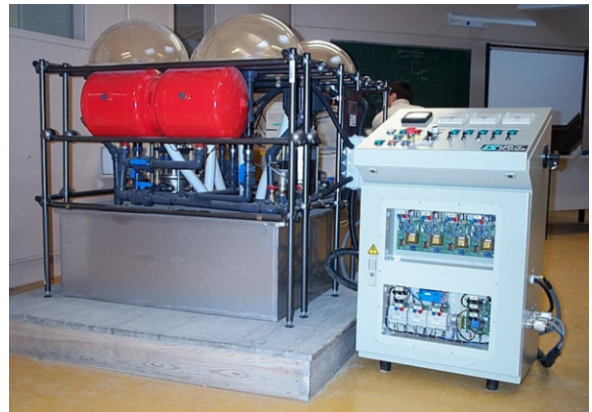


Fig 1: Overview of the irrigation station by sprinkling.

II. IDENTIFICATION AND MODELING OF THE IRRIGATION STATION

The implementation of a mathematical model of a complex real process operating in a stochastic environment draw the attention of many researchers in various disciplines of science and technology. In this context the use of traditional methods of modeling and identification in order to estimate the parameters of such a type of process cannot satisfy the desired performance indices (speed, accuracy and stability). To

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overcome this problem, other techniques such as fuzzy logic and more particularly the T-S fuzzy model showed a good result in the identification of these processes types.

A. Fuzzy coalescence algorithms for system identification

Let us consider a system described by the following differential equation:

$$y(k) = f_{NL}(x_k) \tag{1}$$

with x_k represent the observation vector, $x_k \in R^n$.

The most used algorithms of fuzzy coalescence for the identification parameters of system I are as follows:

- The algorithm of the fuzzy C-averages, or fuzzy c-Means (FCM) ([11]; [13]),
- The algorithm of Gustafson-Kessel (GK) [12],
- The NRFCM algorithm [14].

All these algorithms are based on their minimization of a function objectifies form [15]:

$$J(X, U, V) = \sum_{k=1}^N \sum_{i=1}^c (\mu_{ik})^m (x_k - v_i)^T M (x_k - v_i) \tag{2}$$

where:

$X = \{x_k / k = 1, 2, \dots, N\}$, such that N donate the number of observations;

$U = [\mu_{ik} \in [0, 1]^{(c \times N)}]$, the fuzzy partition matrix of data vector X:

with

$$\sum_{i=1}^c \mu_{ik} = 1 \quad 1 \leq i \leq c \tag{3}$$

V: The prototype clusters vector,

$V = \{v_1, v_2, \dots, v_c\}$, where c represents the rule number (or of clusters) and $v_i \in R^n$.

m: represent the weighting degree.

This parameter influences directly on the form of cluster in data space. Indeed, when m is close to 1, the function of the membership of each cluster becomes al-most Boolean i.e. $\mu_{ik} \in \{0, 1\}$. Whereas when m becomes very large, the partition becomes fuzzier and $\mu_{ik} = 1/c$.

Generally m is selected between 1.5 and 2.5 but in several applications, it is selected between 2 and 4.

In the following section, we present the fuzzy c-means algorithm.

B. Fuzzy c-means (FCM) algorithm

This method is based on minimization of the criterion (4) obtained from the criterion (2) by the addition of the standardization constraint (3) [4].

$$J(X, U, V) = \sum_{k=1}^N \sum_{i=1}^c (\mu_{ik})^m (x_k - v_i)^T M (x_k - v_i) + \sum_{k=1}^N \lambda_k \left[\sum_{i=1}^c \mu_{ik} - 1 \right] \tag{4}$$

In this case the minimization of the criterion (4) can be solved by cancelling the derivative of J where the variables are U, V and λ .

The solution of this criterion is given by:

$$v_i = \frac{\sum_{k=1}^N (\mu_{ik})^m \cdot x_k}{\sum_{k=1}^N (\mu_{ik})^m} \tag{5}$$

$$\mu_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{d_{ik}}{d_{jk}} \right)^{\frac{2}{m-1}}}$$

where

d_{ik} : represent the distance enters X_k and v_i

$$d_{ik} = (x_k - v_i)^T M (x_k - v_i) \tag{6}$$

M: generally selected equal to the identity. The prototype vector of the clusters is given by:

$$d_{ik} = (x_k - v_i)^T M (x_k - v_i) \tag{7}$$

The iteration count of c-means algorithm is selected according to the precise details required by the expert and according to the type of application considered. The criterion of the stop is selected by satisfying the following condition:

$$\|U^{(l)} - U^{(l-1)}\| < \delta \tag{8}$$

where l is the iteration count.

Fuzzy c-means algorithm (FCM):

Being given a whole of data X, FCM algorithm is described by the following stages:

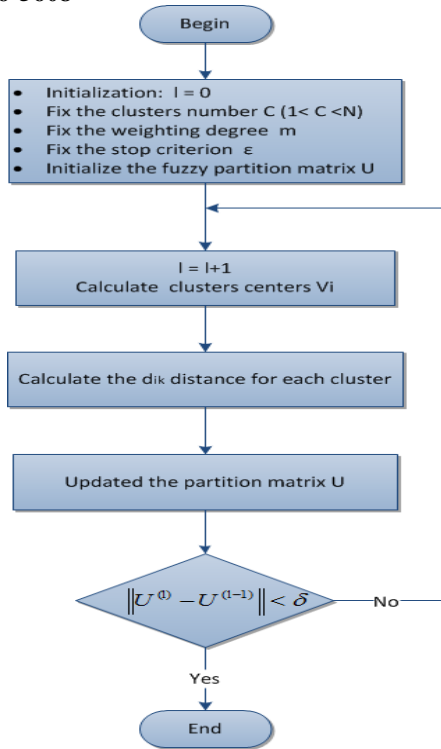


Fig 2: Fuzzy c-means algorithm (FCM).

The FCM Algorithm converges in general towards a local minimum of the objective function. Its performance depends on several factors such as:

- The cluster number;
- Choice m;
- Choice of stop criterion δ .

C. Application of FCM algorithm on the station of irrigation by sprinkling:

Let us consider a system described by the equation (1). Firstly, we approximate the nonlinear function equation (1) by the model of Takagi-Sugeno (TS):

$$R^i : \text{if } x_{k1} \text{ is } A_{i1} \text{ and } x_{k2} \text{ is } A_{i2} \text{ and...} \quad (9)$$

$$\text{and } x_{kn} \text{ is } A_{in} \text{ then } y^i = a_i^T x_k + b_i$$

To represent the rule, we need use observations vector $x_k = [x_{k1}, x_{k2}, \dots, x_{kn}]^T$ and the units fuzzy $A_{i1}, A_{i2}, \dots, A_{in}$ to identify the parameters in the model (9), we builds the matrix of regression X and the vector of the output Y starting from measurements resulting from the system such as:

$$X = [x_1^T, x_2^T, \dots, x_N^T]^T \text{ and } Y = [y_1, y_2, \dots, y_N]^T$$

with $N \gg n$.

The identification of T-S model parameters requires a taking away of the real sig-nals of irrigation station. Using a numerical oscilloscope, we took the real dynamics of pressure and flow of the station of irrigation by sprinkling, then:

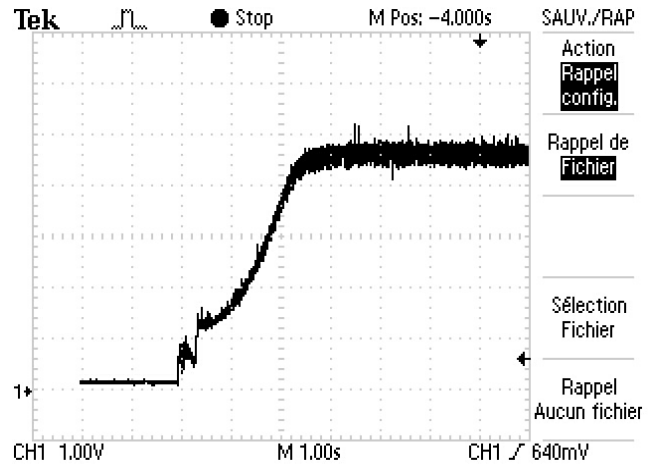


Fig 3: Real curve of the pressure evolution.

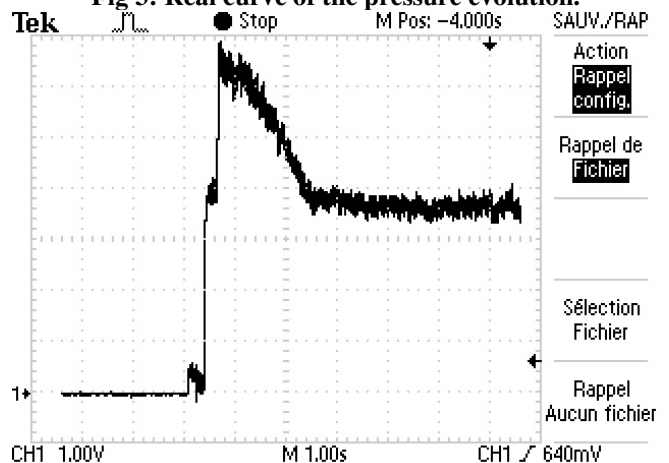


Fig 4: Real curve of the flow evolution.

In order to initialize the iteration count $l=0$, we fix the weighting degree $m=2.75$ what makes it possible to initialize the partial random matrix U. We pass then to the choice of the number of clusters. We apply the classification entropy test CE for each outputs pressure (P) and flow (Q). We noted CEP CEQ respectively.

$$C_{ec}(c) = \frac{1}{N} \sum_{k=1}^N \sum_{i=1}^c \mu_{ik} \log(\mu_{ik}) \quad (10)$$

$$C_{opt} = \min [C_{ec}(c)]$$

	C=2	C=3	C=4	C=5
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$C_{ecP} (10^{-6})$	-0.491	-5.21	-0.41	-1.18
$C_{ecQ} (10^{-6})$	-5.4	-10.8	-3.35	-3.58

The excitation signal of the FCM algorithm is given:

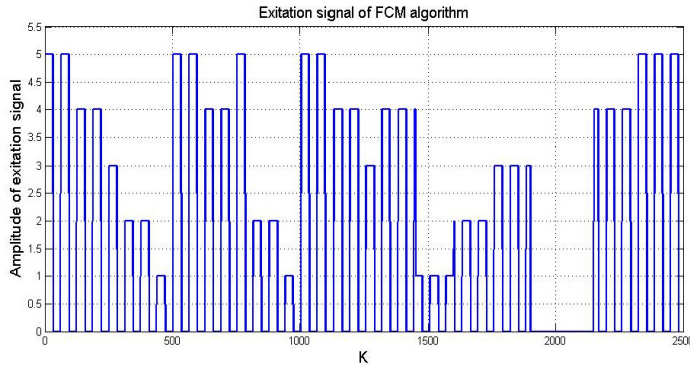


Fig 5: Excitation signal of FCM algorithm.

The excitation signal must be rich to run the system in all operating region. In order to reach all steps, the simulation results of the FCM algorithm are given by the following figures:

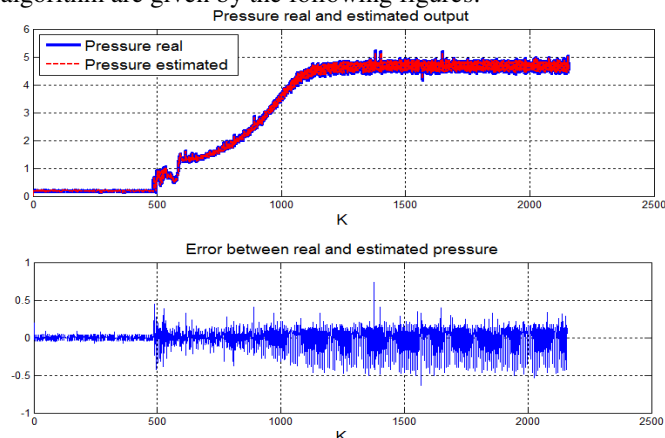


Fig 6: Simulation results of FCM algorithm for the pressure output.

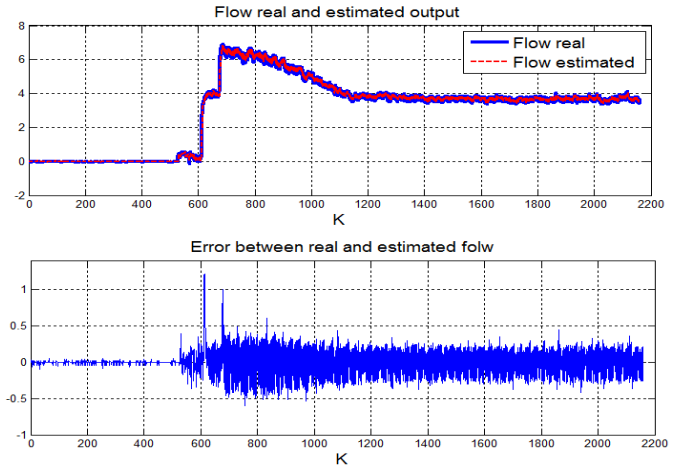


Fig 7: Simulation results of FCM algorithm for the flow output.

Algorithm FCM is followed the real data input of pressure and flow. It is noticed that the error between the evolution of the real and estimated pressure is almost null even for flow. The station of irrigation by sprinkling made up of two nonlinear systems in the same way input and different output, one of pressure and the other of flow, each one partitioned in 3 subsystems. We obtain the following results:

➤ For the pressure sub-systems:

$$\left\{ \begin{array}{l} R_{p1} : y_{p1}(k) = 1.0853y_p(k-1) - 0.1744y_p(k-2) \\ \quad + 0.0570u(k-1) + 0.0318u(k-2) \\ R_{p2} : y_{p2}(k) = 1.0851y_p(k-1) - 0.1743y_p(k-2) \\ \quad + 0.0565u(k-1) + 0.0320u(k-2) \\ R_{p2} : y_{p2}(k) = 1.0852y_p(k-1) - 0.1750y_p(k-2) \\ \quad + 0.0560u(k-1) + 0.0315u(k-2) \end{array} \right.$$

➤ For the flow sub-systems:

$$\left\{ \begin{array}{l} R_{Q1} : y_{Q1}(k) = 1.0853y_Q(k-1) - 0.1744y_Q(k-2) \\ \quad + 1.4118u(k-1) - 1.31u(k-2) \\ R_{Q1} : y_{Q1}(k) = 1.0851y_Q(k-1) - 0.1743y_Q(k-2) \\ \quad + 1.4116u(k-1) - 1.33u(k-2) \\ R_{Q1} : y_{Q1}(k) = 1.0852y_Q(k-1) - 0.1750y_Q(k-2) \\ \quad + 1.4120u(k-1) - 1.31u(k-2) \end{array} \right.$$

For the total identification of system we can draw a rule for each subsystem (flow and pressure) as being modeling and linearization of the whole system, through in-termidiary of the equation (11):

$$y(k+1) = \frac{\sum_{i=1}^c \mu_{ik} \cdot (x(k)) \cdot y_i(k+1)}{\sum_{i=1}^c \mu_{ik} \cdot (x(k))} \quad (11)$$

then

➤ For the pressure output:

$$R_{PG} : y_{PG}(k) = 1.0851y_p(k-1) - 0.1745y_p(k-2) + 0.0563u(k-1) + 0.0317u(k-2) \quad (12)$$

➤ For the flow output:

$$R_{QG} : y_{QG}(k) = 1.0851y_Q(k-1) - 0.1745y_Q(k-2) + 1.4116u(k-1) - 1.32u(k-2) \quad (13)$$

Thus, the open loop transfer functions are:

$$\begin{cases} H_{BOP} = \frac{0.05632z + 0.0317}{z^2 - 1.0851z + 0.1745} \\ H_{BOQ} = \frac{1.4116z - 1.32}{z^2 - 1.0851z + 0.1745} \end{cases} \quad (14)$$

The discrete state representation associated with system (14):

$$\begin{cases} \begin{bmatrix} P_{k+1} \\ Q_{k+1} \end{bmatrix} = \begin{bmatrix} 0.1422 & -0.4403 \\ 0.0917 & 0.9428 \end{bmatrix} \begin{bmatrix} P_k \\ Q_k \end{bmatrix} + \begin{bmatrix} 0.0917 \\ 0.0119 \end{bmatrix} u_k \\ y_k = \begin{bmatrix} 0 & 4.7235 \\ 14.7535 & 4.9165 \end{bmatrix} \begin{bmatrix} P_k \\ Q_k \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_k \end{cases} \quad (15)$$

We introduce the delay $\tau=5s$ into the model obtained,

The system sampling period is chosen $T_e=0.2s$ then the delay $\tau=5s$ is calculated at field discrete time

$$\text{by } z^{-\frac{\tau}{T_e}} = z^{-\frac{5}{0.2}} = z^{-25}.$$

The system (14) becomes:

$$\begin{cases} H_{BOP} = z^{-25} \frac{0.05632z + 0.0317}{z^2 - 1.0851z + 0.1745} \\ H_{BOQ} = z^{-25} \frac{1.4116z - 1.32}{z^2 - 1.0851z + 0.1745} \end{cases} \quad (16)$$

➤ Validation tests of T-S model:

- Root Mean Square Error test (RMSE) [15]:

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^N (y_k - \hat{y}_k)^2} \quad (17)$$

- Variance accounting for test (VAF) [15]:

$$VAF = 100\% \left[1 - \frac{\text{var}(y - \hat{y})}{\text{var}(y)} \right] \quad (18)$$

➤ Variance accounting for test (VAF):

- Root Mean Square Error test (RMSE):

$$\begin{cases} RMSE_{pressure} = 0.1471 \\ RMSE_{flow} = 0.1926 \end{cases} \quad (17)$$

- Variance accounting for test (VAF):

$$\begin{cases} VAF_{pressure} = 99.6090\% \\ VAF_{flow} = 99.3272\% \end{cases} \quad (18)$$

A comparison was made between the estimated outputs and actual outputs collected using a digital oscilloscope.

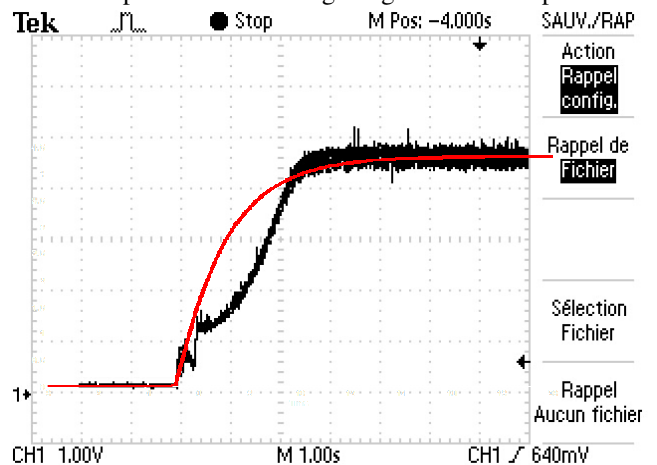


Fig 8: Comparison between measured and estimated pressure.

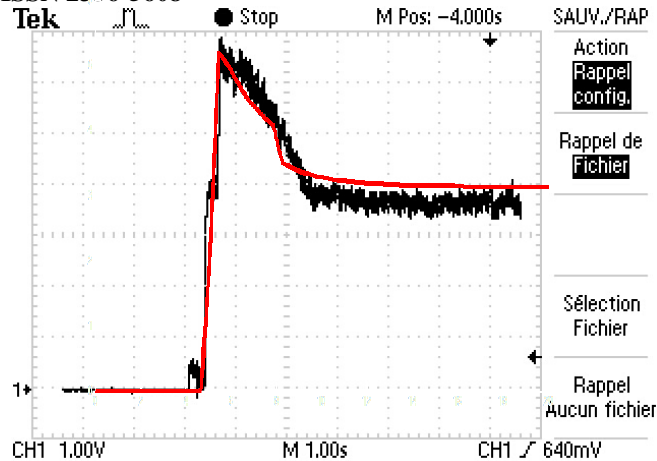


Fig 9: Comparison between measured and estimated flow.

III. CONCLUSION

In this work, we have applied the Takagi- Sugeno algorithm to a station of irrigation with sprinkling (real pumping station) and obtained real values from the station. The system is taken as a black box with outputs pressure and flow. We have modeled and identified the system by the FCM algorithm.

After obtained the T-S model we have validated curves is almost identical to the real ones. The obtained linear model gives a good description of the behavior of the system and the importance of the clustering methods.

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